Exercícios Aula 1 (Sistemas de Unidades)

3.8.1

temos que saber que 1 utm = 9,8 kg não esquecer de elevar os números na conversão, como na letra c) e d)

a) $[\text{Joule}]_{S.I.}$ e $[\text{kgm}]_{S.T.}$ $\quad \because [E] = [F][d] = MLT^{-2} \cdot L = ML^2T^{-2}$

S.I. → $\frac{\text{Joule}}{\text{kgm}} = \frac{\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}}{\text{utm} \cdot \text{m}^2 \cdot \text{s}^{-2}} = \frac{\text{kg}}{\text{utm}} = \frac{\text{kg}}{9{,}8\,\text{kg}}$, Logo $1\ \text{Joule} = \frac{1}{9{,}81}\ \text{kgm}$ ou **1 kgm = 9,81 Joule**

S.T. →

b) $[\text{kgf}]_{S.T.}$ e $[\text{N}]_{S.I.}$ $\quad \because [F] = [m] \cdot [a] = MLT^{-2}$

S.I. → $\frac{\text{N}}{\text{kgf}} = \frac{\text{kg} \cdot \text{m} \cdot \text{s}^{-2}}{\text{utm} \cdot \text{m} \cdot \text{s}^{-2}} = \frac{\text{kg}}{\text{utm}} = \frac{\text{kg}}{9{,}8\,\text{kg}} = \frac{1}{9{,}8}$, Logo $1\ \text{N} = \frac{1}{9{,}8}\ \text{kgf}$ ou **1 kgf = 9,8 N**

S.T. →

c) $[\text{Pascal}]_{S.I.}$ e $[\text{Bazia}]_{CGS}$ $\quad [P] = \frac{F}{A} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

S.I. → $\frac{\text{Pa}}{\text{b}} = \frac{\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}}{\text{g} \cdot \text{cm}^{-1} \cdot \text{s}^{-2}} = \frac{10^3\,\text{g} \cdot (10^2)^{-1}\,\text{cm}^{-1} \cdot \text{s}^{-2}}{\text{g} \cdot \text{cm}^{-1} \cdot \text{s}^{-2}} = 10^3 \cdot 10^{-2} = 10$ ∴ Logo, **1 Pascal = 10 Bazia**

CGS →

d) $[\text{kgm/s}]_{S.T.}$ e $[\text{Erg/s}]_{CGS}$ $\quad [\text{Pot}] = \frac{E}{t} = \frac{MLT^{-2} \cdot L}{T} = ML^2T^{-3}$

S.T → $\frac{\text{kgm/s}}{\text{Erg/s}} = \frac{\text{utm} \cdot \text{m}^2 \cdot \text{s}^{-3}}{\text{g} \cdot \text{cm}^2 \cdot \text{s}^{-3}} = \frac{9{,}81 \cdot 10^3\,\text{g} \cdot (10^2)^2\,\text{cm}^2}{\text{g} \cdot \text{cm}^2} = 9{,}81 \cdot 10^3 \cdot 10^4 = 9{,}81 \cdot 10^7$ ∴ Logo, **1 kgm/s = $9{,}81 \cdot 10^7$ Erg/s**

CGS →

3.8.2

$\text{Pot} = 4 \cdot \rho \cdot V^3 \cdot R^2$ (Princípio da Homogeneidade)

$[\text{Pot}] = \frac{E}{s} = \frac{F \cdot \ell}{s} = \frac{m \cdot a \cdot \ell}{s} = \frac{MLT^{-2} \cdot L}{T} = ML^2T^{-3}$

$[4] = 1$

$[\rho] = \frac{\text{massa}}{\text{volume}} = \frac{M}{L^3} = ML^{-3}$ ∴ Logo, $1^\circ = ML^2T^{-3}$ ∴ $ML^2T^{-3} = M\!\not{L}^{3} \cdot L^{-3}T^{-3}L^2$ ∴ **$ML^2T^{-3} = ML^2T^{-3}$**

$[V] = \frac{\Delta X}{\Delta t} = LT^{-1}$ $\quad 2^\circ = 1 \cdot ML^{-3} \cdot (LT^{-1})^3 \cdot L^2$ dimensionalmente verdadeira!

R = Raio = L

3.8.3

$\text{Pot} = \rho \cdot g \cdot H \cdot Q$, onde $[\text{Pot}] = \frac{E}{s} = \frac{F \cdot \ell}{s} = \frac{MLT^{-2} \cdot L}{T} = ML^2T^{-3}$

$[\rho] = \frac{\text{massa}}{\text{volume}} = \frac{M}{L^3} = ML^{-3}$

$[g]$ = aceleração = $\frac{\Delta V}{\Delta t} = LT^{-2}$ ∴ $ML^2T^{-3} = M\!\not{L}^{3} \cdot LT^{-2} \cdot L \cdot \not{L}T^{-1}$

$[H] = L$ $\quad$ **$ML^2T^{-3} = ML^2T^{-3}$**

$[Q]$ = vazão = $\frac{\text{volume}}{\text{tempo}} = \frac{L^3}{T} = L^3T^{-1}$ dimensionalmente legítima!

3.8.3.1

$S = S_0 + V_0 \cdot t$ ∴ Logo, $L = L \cdot LT^{-1} \cdot T$ ∴ $L = L$

$[S_0] = [S] = L$ dimensionalmente legítima!

$[V_0] = LT^{-1}$

$[t] = T$

3.8.4

$V = \sqrt{\frac{1}{\mu \cdot S}}$, onde $V = \frac{\Delta x}{\Delta t} = LT^{-1}$

$[\mu]$ = permeabilidade magnética $\Rightarrow F = B \cdot i \cdot \ell$, $B = \frac{\mu \cdot i}{2\pi \cdot d}$ ∴ $\mu = \frac{B \cdot 2\pi \cdot d}{i}$ ∴ $\mu = \frac{F}{i\ell} \cdot \frac{2\pi \cdot d}{i} = \frac{MLT^{-2} \cdot L}{Q \cdot T^{-1} \cdot L \cdot Q \cdot T^{-1}} = MLQ^{-2}$

$[S]$ = permetividade magnética $\Rightarrow F = \frac{1}{4\pi S} \cdot \frac{Q_1 Q_2}{d^2}$ ∴ $F(4\pi S d^2) = Q_1 Q_2$ ∴ $S = \frac{Q_1 Q_2}{4\pi \cdot F \cdot d^2} = \frac{Q^2}{1 \cdot MLT^{-2} \cdot L^2} = M^{-1} L^{-3} T^2 Q^2$

$\mu \cdot S = MLQ^{-2} \cdot M^{-1} L^{-3} T^2 Q^2 = L^{-2} T^2$ ∴ $\sqrt{\frac{1}{\mu \cdot S}} = \sqrt{L^2 T^{-2}} = \sqrt{(LT^{-1})^2} = LT^{-1}$ dimensionalmente legítima!

3.8.5

$F = \frac{G(M_1 \cdot M_2)}{d^2}$ ∴ $G = \frac{F \cdot d^2}{M_1 M_2}$ ∴ $[G] = \frac{MLT^{-2} \cdot L^2}{M \cdot M} = M^{-1} L^3 T^{-2} = \frac{L^3}{M \cdot T^2}$

$[F] = MLT^{-2}$

$[M_1] = [M_2] = M$ ∴ Em S.I.: $[G] = \frac{m^3}{kg \cdot s^2}$

$[d_2] = L$ Em S.T.: $[G] = \frac{m^3}{utm \cdot s^2}$

Em CGS: $[G] = \frac{cm^3}{g \cdot s^2}$

4.2.1

$F = f(m, V, R)$, $K = 1,2$ ∴ $F = 1,2 \cdot [m]^a \cdot [V]^b \cdot [R]^c$

$[F] = MLT^{-2}$ $MLT^{-2} = 1 \cdot M^a \cdot (LT^{-1})^b \cdot L^c$ ∴ $MLT^{-2} = M^a \cdot L^b T^{-b} \cdot L^c$ ∴ $M^1 L^1 T^{-2} = M^a \cdot L^{b+c} \cdot T^{-b}$

$[m] = M$

$[R] = L$

$[V] = LT^{-1}$

$\begin{cases} a = 1 \\ b + c = 1 \therefore c = 1 - b = 1 - 2 \therefore c = -1 \\ -b = -2 \therefore b = 2 \end{cases}$

Logo, $F = 1,2 \cdot m^1 \cdot V^2 \cdot R^{-1}$ ∴ $F = 1,2 \frac{mV^2}{R}$

$[k = 1,2] = 1$

4.2.2

$R = f(V, q, m, B)$, $K = 1,7$ ∴ $R = 1,7 \cdot [V]^a \cdot [q]^b \cdot [m]^c \cdot [B]^d$

$[R] = L$ $L = (LT^{-1})^a \cdot Q^b \cdot M^c \cdot (MQ^{-1}T^{-1})^d$ ∴ $L = L^a T^{-a} \cdot Q^b \cdot M^c \cdot M^d \cdot Q^{-d} \cdot T^{-d}$

$[V] = LT^{-1}$ $L^1 \cdot T^0 \cdot M^0 \cdot Q^0 = L^a \cdot T^{-a-d} \cdot M^{c+d} \cdot Q^{b-d}$

$[q] = Q$

$[m] = M$

$[B] = ?$ ∴ $F = B \cdot i \cdot \ell$, sendo $[i] = \frac{Q}{T}$

$\begin{cases} a = 1 \\ -a - d = 0 \therefore d = -a \therefore d = -1 \\ c + d = 0 \therefore c = -d \therefore c = 1 \end{cases}$

$R = 1,7 \cdot [V]^a \cdot [q]^b \cdot [m]^c \cdot [B]^d$

$R = 1,7 \cdot V^1 \cdot q^{-1} \cdot m^1 \cdot B^{-1}$

$[B] = \frac{F}{i \cdot l} = \frac{MLT^{-2}}{QT^{-1} \cdot L} = MQ^{-1}T^{-1}$ //

$\begin{cases} b - d = 0 \therefore b = d \therefore b = -1 \end{cases}$ //

$R = 1{,}7 \cdot \frac{Vm}{9 \cdot B}$ //

4.2.3

$T = f(l, g) \therefore T = K \cdot [L]^a \cdot [LT^{-2}]^b \therefore T^1 = L^a \cdot L^b \cdot T^{-2b} \therefore L^0 \cdot T^1 = L^{a+b} \cdot T^{-2b}$

$[T]$ = segundo = T

$[l]$ = L

$[g]$ = aceleração = LT^{-2}

$\begin{cases} a + b = 0 \therefore a = -b \therefore a = \frac{1}{2} \\ -2b = 1 \therefore b = -\frac{1}{2} \end{cases}$ //

$T = K \cdot [l]^a [g]^b = K \cdot l^{1/2} \cdot g^{-1/2} = K \cdot \frac{l^{1/2}}{g^{1/2}} = K\left(\frac{l}{g}\right)^{1/2} \therefore T = K\sqrt{\frac{l}{5}}$ //

Exercícios Aula 2 - Fundamentos da Mecânica dos Fluidos

5.9.5.3

$V = 1\,m^3$, $m = 1000\,kg$, $g = 9{,}81\,m/s^2$

densidade específica / massa específica (ρ) = $\frac{massa}{Volume}$ $\therefore$ $\rho = \frac{1000\,kg}{1\,m^3} = 1000\,kg/m^3$ // ($\gamma = 9810\,utm/m^3$)

Peso específico (γ) = $\frac{Peso}{Volume} = \frac{massa \cdot gravidade}{Volume}$ $\therefore$ $\gamma = \frac{1000\,kg \cdot 9{,}81\,m/s^2}{1\,m^3} = 9810\,N/m^3$ // ($\rho = 1000\,kgf/m^3$)

5.9.5.5

$F = 8\,kgf$, $V = 9{,}639\,cm^3$ $\quad 1\,kgf = 9{,}81\,N \quad 1\,kgf = 1\,kg \quad 1\,cm^3 = (10^{-2})^3\,m^3 = 10^{-6}\,m^3$

a) massa específica (ρ) = $\frac{m}{V}$ $\therefore$ $\rho = \frac{8}{9{,}639 \cdot 10^{-6}}\,kg/m^3 = 0{,}8299616 \cdot 10^6\,kg/m^3$ // (S.I.)

Peso específico (γ) = $\frac{m \cdot a}{V}$ $\therefore$ ρ em S.I = γ em S.T $\quad \gamma = 0{,}8299616 \cdot 10^6\,kgf/m^3$ // (S.T.)

Volume específico (v) = $\frac{V}{m}$ $\therefore$ $v = \frac{9{,}639 \cdot 10^{-6}}{8}\,m^3/kg = 1{,}204875 \cdot 10^{-6}\,m^3/kg$ (S.I.)

b) Densidade Relativa (D) = $\frac{\rho\,Óleo}{\rho\,H_2O} = \frac{\gamma\,Óleo}{\gamma\,H_2O}$, onde $\rho_{H_2O} = 1000\,kg/m^3$ e $\gamma_{H_2O} = 1000\,kgf/m^3$ $\therefore$ $D = \frac{0{,}8299616 \cdot 10^6\,kg/m^3}{10^3\,kg/m^3} \cong 830$ //

$D \cong 830$ (Adimensional)

5.9.5.6

$\tau = \mu \frac{du}{dy}$, onde $\tau = N/m^2$, $\frac{du}{dy} = \frac{m/s}{m} = \frac{1}{s}$, logo $\mu = \left[\frac{N \cdot s}{m^2}\right]$ //

$\rho = \left[\frac{kg}{m^3}\right]$ //

$v = \frac{\mu}{\rho} = \frac{N \cdot s \cdot m^{-2}}{kg \cdot m^{-3}} = \frac{kg \cdot m \cdot s^{-2} \cdot s \cdot m^{-2}}{kg \cdot m^{-3}} = m^2 s^{-1}$, logo $v = \left[\frac{m^2}{s}\right]$ //

Viscosidade cinemática (v) = $0{,}033\,m^2/s$, $D = 0{,}86$, $\gamma_{H_2O} = 1000\,kgf/m^3$

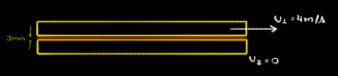
$v = \frac{\mu}{\rho}$, $D = \frac{\gamma_f}{\gamma_{H_2O}} = \frac{\rho_f}{\rho_{H_2O}}$ $\therefore$ $0{,}86 = \frac{\gamma_f}{10^3}$ $\therefore$ $\gamma_f = 0{,}86 \cdot 10^3 = 860\,kgf/m^3$ // $\longrightarrow \rho_f = 860\,kg/m^3$ //

$\frac{S.I.}{S.T} = \frac{kg \cdot m^{-3}}{utm \cdot m^{-3}} = \frac{kg}{9{,}81\,kg} = \frac{1}{9{,}81}$, Logo $1\,kg/m^3 = \frac{1}{9{,}81}\,utm/m^3$ $\therefore$ $\rho_f = 860 \cdot \frac{1}{9{,}81} = 87{,}665\,utm/m^3$ //

kgf

$\mu = \upsilon \cdot \rho = 0{,}033 \cdot 87{,}665 \ \frac{m^2}{s} \cdot \frac{utm}{m^3} \therefore \mu = 2{,}8929 \ \frac{utm}{m \cdot s} = 2{,}8929 \ \frac{utm \cdot m}{s^2} \cdot \frac{1}{m^2} \therefore \mu = 2{,}8929664 \ \frac{kgf \cdot s}{m^2}$ //

S.9.S.7



Propriedades do óleo: $\upsilon = 0{,}15$ Stokes $\longrightarrow \upsilon = 0{,}15 \ cm^2/s = 0{,}15 \cdot (10^{-2})^2 \ m^2/s = 0{,}15 \cdot 10^{-4} \ m^2/s$

$\rho = 905 \ kg/m^3$

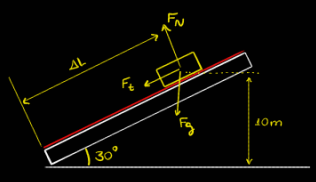
$\tau = \mu \cdot \frac{du}{dy}$

$\upsilon = \frac{\mu}{\rho} \therefore \mu = \upsilon \cdot \rho \longrightarrow \mu = 0{,}15 \cdot 10^{-4} \cdot 905 = 0{,}013575 \ \frac{N \cdot s}{m^2}$ //

$du = 4 m/s$ // $\quad dy = 3 mm = 3 \cdot 10^{-3} m$ //

$\tau = 0{,}013575 \cdot \frac{4}{3 \cdot 10^{-3}} = 18{,}1 \ N/m^2$ ou $\tau = 18{,}1 \ Pa$ //

S.9.S.8



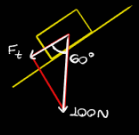
placa retangular de 4m por 5m $\longrightarrow$ Área $= 5 \cdot 4 = 20 m^2$

$dy = 1 mm = 10^{-3} m$

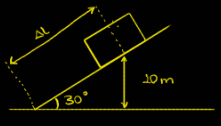
$\mu = 0{,}01 \ N \cdot s/m^2$

Peso da placa $= 100 N$

Peso $= F_g = 100 N$



$\frac{ca}{H} = \cos\alpha \therefore \cos 60° = \frac{F_t}{100} \therefore F_t = 100 \cdot \cos 60° = 50 N$ //



$\therefore \frac{co}{H} = \sin\alpha \therefore \sin 30° = \frac{10}{\Delta L} \therefore \Delta L = \frac{10}{\sin 30°} = 10 \cdot \frac{2}{1} = 20 m$ //

$\tau = \mu \cdot \frac{du}{dy} \quad \therefore du = \frac{\tau \cdot dy}{\mu}$, como o plano está parado, $du = U_0$ que é a velocidade do bloco

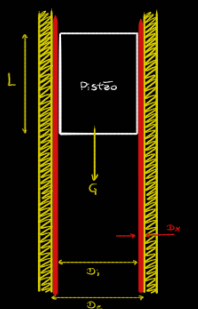
Sabendo que a tensão de cisalhamento é uma força paralela ao plano dividida pela área $\therefore \tau = \frac{F_t}{A}$

$U_0 = \frac{F_t \cdot dy}{\mu \cdot A} = \frac{50 \cdot 10^{-3}}{0{,}01 \cdot 20} \therefore U_0 = 0{,}25 \ m/s$ //

$U_0 = \frac{\Delta x}{\Delta t} \therefore \Delta t = \frac{\Delta x}{U_0} = \frac{20}{0{,}25} = 80 s$ //

Exercício Proposto 2

$G = 10N$; $L = 0{,}08 m$; $V = 3 m/s$; $De = 0{,}101 m$; $Di = 0{,}1 m$; $m_{óleo} = 0{,}01 kg$.



Distância entre cilindro e o pistão $(D_x) = \frac{De - Di}{2} = \frac{0{,}001}{2} = 0{,}0005 = 5 \cdot 10^{-4} m$ //

Primeiro encontramos o volume da camada de óleo. Sabendo que $V_{cilindro} = \pi r^2 h$, temos:

$V_{óleo} = \left[\pi \cdot \left(\frac{De}{2}\right)^2 \cdot h\right] - \left[\pi \cdot \left(\frac{Di}{2}\right)^2 \cdot h\right] = \pi \cdot h \left(\frac{De^2 - Di^2}{4}\right) = \frac{\pi h}{4}(De^2 - Di^2) = \frac{\pi \cdot 0{,}08}{4}(0{,}101^2 - 0{,}1^2) = 1{,}269 \cdot 10^{-5} \ m^3$ //

Com isso, podemos encontrar a densidade / massa específica do fluído:

$$\rho = \frac{m_{óleo}}{V_{óleo}} = \frac{0,01}{1,269 \cdot 10^{-5}} = 791,8 \ kg/m^3 \ //$$

A tensão de cisalhamento é: $\tau = \frac{F}{A} = \mu \frac{dv}{dy}$, onde $dV = V$; $dy = Dx$; $F = G$.

A área referente ao cisalhamento não é a área na tampa do cilindro, mas sim na área das paredes laterais, onde ocorre essa força de arrasto.

E temos a $A_{lateral}$ como o perímetro vezes a altura. Logo, $A = 2\pi R \cdot h = \pi \cdot D \cdot h \ \therefore \ A = \pi \cdot D_i \cdot L$

Isolando μ, temos: $\mu = \frac{F}{A} \cdot \frac{dy}{dV} = \frac{G \cdot Dx}{\pi D_i L \cdot V} = \frac{10 \cdot 5 \cdot 10^{-4}}{\pi \cdot 0,1 \cdot 0,08 \cdot 3} = 0,0663 \ \frac{N \cdot s}{m^2} \ //$

Viscosidade cinemática $(\nu) = \frac{\mu}{\rho} = \frac{0,0663}{791,8} = 8,3 \cdot 10^{-5} \ \frac{\frac{N \cdot s}{m^2}}{\frac{kg}{m^3}} = \frac{N \cdot s}{m^2} \cdot \frac{m^3}{kg} = \frac{N \cdot s \cdot m}{kg} = N \cdot \frac{s \cdot m}{kg} = \cancel{kg} \cdot \frac{m}{s^2} \cdot \frac{s \cdot m}{\cancel{kg}} = m^2/s \ //$

$\nu = 8,3 \cdot 10^{-5} \ m^2/s \ //$

→ Ou $8,3 \cdot 10^{-5} \cdot (10^2)^2 \ \frac{cm^2}{s} = 8,3 \cdot 10^{-1} = 0,83 \ cm^2/s$ em CGS // É como $1 \ cm^2/s = 1$ stoke $\therefore$ $\nu = 0,83 \ St \ //$